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MANIFOLD HOMEOMORPHISM GROUP

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Annotation: *this article shows that the group* $Homeo_F(M)$ *of homeomorphisms of the manifold* $Homeo(M)$ *is a topological subgroup in the compactly open topology* M.

***Keywords*:** continuous maps, homeomorphism, topological manifold, world manifold, map, atlas, smooth atlas, maximal atlas, smooth structural manifolds, smooth manifolds, compact open topology.

One of the important reflections for us within continuous reflections is topological reflection. Topological reflection is also known as homeomorph reflection.

X and Y topological spaces, $f: X \to Y-$ let the reflection be given. If f reverse reflection to reflection f^{-1} exists and f, f^{-1} when the reflections are continuous, f is called topological reflection or homeomorphism.

As the simplest example of topological reflection $f(x)=x$ the rule defines a particular $f: X \to X$ we can get reflection.

It follows directly from the definition of topological reflection that, if f in the case of topological reflection, this is called inverse reflection f^{-1} will also be topological reflection. Now f for reverse reflection to exist, let us pay attention to the necessary and sufficient condition. Reverse reflection Y to each point of X puts each point of the corresponding. So optional $y \in Y$ one for $x \in X$ available as, $f(x)=y$ equality must be appropriate. For this, the $f(X)=Y$ be, i.e. f the overlap should be reflection. In addition f^{-1} reverse reflection $y \in Y$ point one $x \in X$ point matching $x_1 \neq x_2$ when $f(x_1) \neq f(x_2)$ it is necessary to be, i.e., to be a reciprocal one-valued reflection.

And so, f reverse reflection to f^{-1} to exist f it is necessary and sufficient that there should be a superimposed and mutually one-valued reflection. If X and Y for topological spaces $f: X \to Y$ if there is a topological reflection, X and Y topological spaces are called mutually homeomorphic or topologically equivalent spaces. Properties of topological spaces that are preserved in topological reflection (that is, transferred from one to another) are called topological properties.

Example 1. $X=(a,b)$, $Y=(c,d)$ to be X and Y topology in spaces R^1 is determined using the topology in That's it $f: X \to Y$ to reflect $f(x)=\frac{d-c}{b-a}(x-a)+c$ if we determine using the formula f is a homeomorphism because f a linear function is continuous and its inverse is also continuous.

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To us M Hausdorff space and φ be a given homeomorphism.

Definition 1. If $M-$ each of the Hausdorff topological spaces $p \in M$ contains this point for the point U open set and $\varphi: U \to G,\ G \subset R^n$, if there is a homeomorphism, then M this is the space (U,φ) together with cards is called a topological manifold. It should be noted here that $U - M$ is an open set in $p \in M,\ \varphi: U \to \varphi(U)-$ homeomorphism and $\varphi(U) - R^n$ is an open set in space. $\{(U,\varphi)\}$ family of cards M is called an atlas of and A is determined by $A = \{(U,\varphi)\}$. Found in this definition n to the number M is called the dimension of the multiplicity and is $\dim(M) = n$ is written in the form.

Example 2. To us $f: R^n \to R^1$ - is a continuous function given by $G_f \left(G_f \subset R^{n+1}\right)$ and the collection $f(x_1, x_2, \ldots, x_n)$ be the graph of the function, i.e

$$G_f = \left\{\left(x_1, x_2, \ldots, x_n, x^{n+1}\right) : x^{n+1} = f(x_1, x_2, \ldots, x_n)\right\}.$$

It is easy to understand G_f space, one $U = G_f$ from a card atlas

found n – will be a dimensional multiplicity.

$M - n$ dimensional topological manifold, A be his atlas. A from $\forall (U,\varphi)$ and (V,ϕ) let's take cards. $U \cap V \neq \varnothing$ let it be. If the formulas for the substitution of coordinates determine this

$$\phi \circ \varphi^{-1} : \varphi(U \cap V) \to \phi(U \cap V) \quad (1)$$

reflection $C^\infty -$ consists of a diffeomorphism, then the cards under consideration $C^\infty -$ called matched cards. (1) since the reflection is a homeomorphism, changing the coordinates to make it a diffeomorphism functions

$$\begin{cases} y^1 = y^1\left(x^1, \ldots, x^n\right) \\ y^2 = y^2\left(x^1, \ldots, x^n\right) \\ \ldots\ldots\ldots\ldots\ldots \\ y^n = y^n\left(x^1, \ldots, x^n\right) \end{cases}$$

$C^\infty -$ belonging to the class and $\det \left\| \frac{\partial y^i}{\partial x^j} \right\| \neq 0$ the sufficiency of being. If (U,φ) va (V,ϕ) for cards $U \cap V \neq \varnothing$ if so, we call them matched cards by definition. Made up of customized cards (or rather $C^\infty -$ made up of aligned cards) the atlas is called a smooth atlas. If A This is the card that is aligned with all the cards of the atlas A if lies at , then A **called maximal atlas**. If M smooth maximal atlas for a topological manifold $A_{\max}$ if available M multiplicity with smooth structure. M This is the majority $A_{\max}$ together with the atlas $(M, A_{\max})$ is called smooth multiplicity.

Example 3. $S^1 -$ is a one-dimensional smooth manifold, i.e

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$$S^1 = M^1,\ S^1 = (x,y) \in R^2,\ x^2 + y^2 = 1.$$

If $M-$ riman multiplicity and $Homeo(M)-$ M the set of all topological reflections in the manifold. If $\varphi_1, \varphi_2 \in Homeo(M)$ and their composition $\varphi_1 \circ \varphi_2$ will be a topological reflection.

If $\varphi_1\varphi_2 = \varphi_1 \circ \varphi_2$ let's say, $\varphi_1, \varphi_2 \in Homeo(M)$ form a group. Here we are $Homeo(M)$ is a compact open topology. Its definition is as follows.

Each of us F which lies in some layer of the layer $\{K_\lambda\}$ the family of all compact sets and M in the majority $\{U_\beta\}$ Let be the family of all open sets given by each $K_\lambda \subset L_\alpha$ va U_β for a couple $f(K_\lambda) \subset U_\beta$ All that is appropriate $f \in Homeo_F(M)$ Let's look at a collection of reflections. This is a collection of reviews

$$[K_\lambda, U_\beta] = \{ f : M \to M \mid f(K_\lambda) \subset U_\beta \}$$

as. This $\sigma_1 = \{[K_\lambda, U_\beta]\}$ after $\sigma_2 = \left\{ \bigcap_{i=1}^{k} [K_{\lambda_i}, U_{\beta_i}] \right\} \cup \varnothing$ we look after the family.

This family forms the basis for some topologies. This topology is layered compact-open or $F-$ we call compact-open topology[3].

To us k sized F_1, F_2 n-dimensional with layers M, N let smooth polynomials be given (here $0 < k < n$).

Definition 2. Someone $\varphi : M \to N$ in homeomorphism F_1 is optional in the layer L_α of the layer $\varphi(L_\alpha)$ the opposite F_2 if it is a layer of a layer, it is given (M, F_1) and (N, F_2) is called homeomorphic and $(M, F_1) \approx (N, F_2)$ is written as.

Given (N, F_2) plurality (N, F_2) reflecting the multitude φ a homeomorphism is called layer-preserving and $\varphi : (M, F_1) \to (N, F_2)$ is written in the form.

If $M = N$ and $F_1 = F_2$ if the relations are appropriate, we say that the homeomorphism of the layered manifold is given.

Example 4. $M = R^2(x,y) - (x,y)$ Euclidean plane with Cartesian coordinates F don't fold L_α layer $y = \alpha = const$ given by Eq. Homeomorphism of the fold plane $\varphi(x,y) = \left(x + y, y^{\frac{1}{3}} \right)$ given by the formula $\varphi : R^2 \to R^2$ homeomorphism.

Collapsible (M, F) the set of all homeomorphisms of the polynomial $Homeo_F(M)$ as.

Theorem 1. $Homeo_F(M)$ group $Homeo(M)$ is a partial group of and is a topological group in a compact open topology.

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