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From The Experience Of Using Innovative Teaching Methods In Teaching The Topic "Wave Function"

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Abstract: The article describes some results of experiments carried out to improve the effectiveness of teaching the topics of wave function for physics students in higher education, some features of innovative forms and methods of teaching are considered.
Keywords: Schrodinger equation, wave function, two-dimensional potential well, probability density, computer modelling.

INTRODUCTION

Quantum mechanics is the foundation of modern physics. Most discoveries in modern physics have been predicted and described on the basis of quantum mechanics.

Unlike other sections of general physics, a course in quantum mechanics is disadvantaged by the practical lack of demonstrations and laboratory work. To improve the effectiveness of teaching the basics of quantum mechanics, from our point of view, it is advisable to apply information technology.

The problem of interpretation of quantum mechanics has existed for almost a hundred years. There are numerous versions of interpretation. This is due to the fact that quantum mechanics is an axiomatic concept. This means that quantum mechanics contains an axiomatic object defined only by its properties. This axiomatic object is the wave function [1].

The purpose of this article is to study the behavior of a particle in a visually created potential well with specified conditions.

LITERATURE AND METHOD

The stationary Schrodinger equation describes the states of the system (in which the energy takes on certain values) [2]:

$$\Delta\psi + \frac{2m}{\hbar^2}[E - U(x,y)]\psi = 0$$

Solutions of this equation ψ_n are called eigenfunctions of the system, and the corresponding values of the parameter E_n are called eigenvalues of energy.

Let us give the solution to the Schrodinger equation for the simplest case of a two-dimensional rectangular well with infinitely high walls (Fig. 1):

$$U(x,y)=\begin{cases}0, & M\in\Omega\\ \infty, & M\notin\Omega\end{cases}\qquad \Omega=\begin{cases}0<x<a\\ 0<y<b\end{cases}\qquad M(x,y)$$

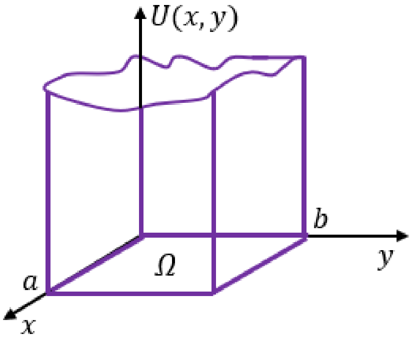

Figure 1. General view of a two-dimensional potential well.

Based on the symmetry of the problem, we can represent the wave function as a product of functions of one variable [2]

$$\psi(x,y)=\psi_x(x)\cdot\psi_y(y)$$

Then the Schrödinger equation takes the form:

$$\frac{d^2\psi_x}{dx^2}+\frac{2m}{\hbar^2}E_{n_x}\psi_x=0$$

Taking into account the boundary conditions $\psi_x(0)=0,\ \psi_x(a)=0,$ by the solution, the equation can be written as

$$\psi_{x,n}=A_x\sin\frac{\pi n_x}{a}x,$$

Similarly, can be assumed for ψ_y. The desired wave function is found as follows:

$$\psi_{n_1,n_2}(x,y)=A\sin\frac{\pi n_x}{a}x\sin\frac{\pi n_y}{b}y\ \ n_x,n_y=1,2,3...$$

RESULTS

To simulate particles in a two-dimensional potential well, a program was developed in Visual Basic with the connection of appropriate libraries capable of simulating the behavior of quantum particles [4,5].

When you start the program, the main program window opens (Fig. 2), in which there is a data input panel at the bottom left, and an output window at the right.

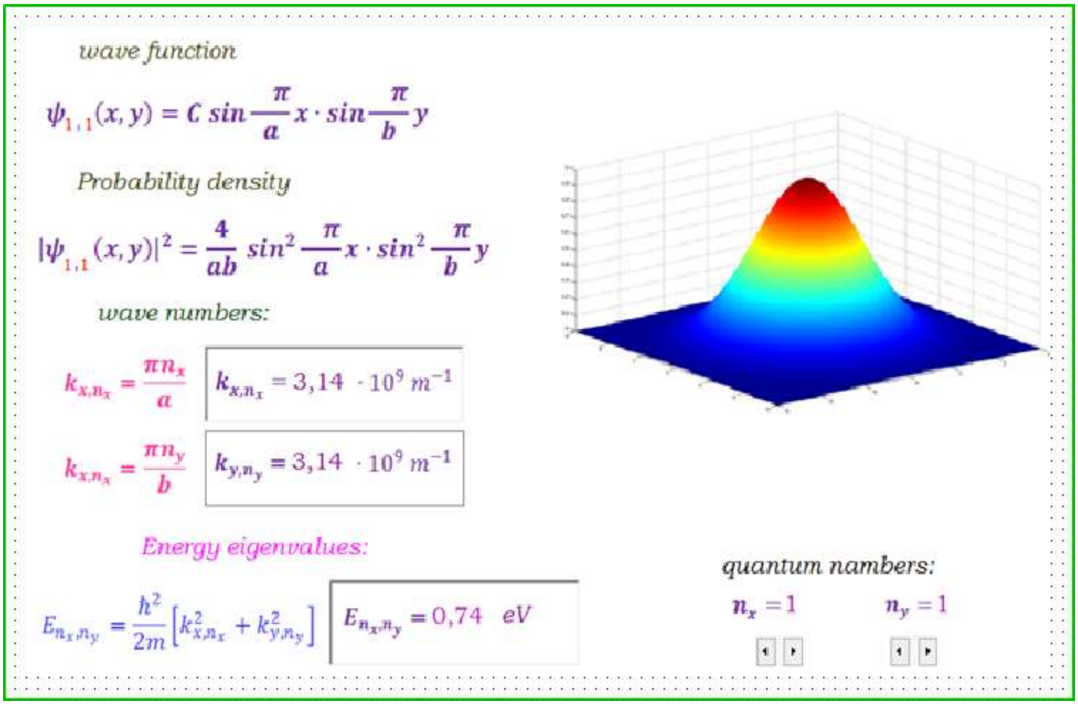

Fig. 2. The main program window with the selected task parameters.

The program can operate in two modes: manual and automatic. In manual mode, the user independently sets the value of the quantum number and the potential well parameter. And on their basis, the program builds graphs of the wave function and probability density for the corresponding energy value E.

There are special fields for displaying the wave number and energy eigenvalues corresponding to the quantum numbers of particles and the dimensions of the potential well.

The probability density of finding a particle in a two-dimensional potential well is determined by the expression:

$$\rho_{n_x,n_y}(x,y) = \left|\psi_{n_x,n_y}(x,y)\right|^2 = \frac{4}{ab}\sin^2\left(\frac{\pi n_x}{a}x\right)\sin^2\left(\frac{\pi n_y}{b}y\right)$$

Consider the examples.

1. Determine the points at which the probability density of finding a particle in a rectangular potential well (b=2a) is maximum, for the case when the quantum numbers of the particle are equal to $n_x = 2$ and $n_y = 1$.

In order to find the maximum value of the function $\rho_{2,1}(x,y)$, we first take the derivative of this function with respect to, and then with respect to, and then equate them to zero [7]:

$$\frac{\partial\rho_{2,1}(x,y)}{\partial x} = \frac{16\pi}{a^3}\sin\left(\frac{2\pi}{a}x\right)\cdot\cos\left(\frac{2\pi}{a}x\right)\cdot\sin^2\left(\frac{\pi}{a}y\right) = 0$$

$$\frac{\partial\rho_{1,2}(x,y)}{\partial y} = \frac{8\pi}{ab^2}\sin^2\left(\frac{2\pi}{a}x\right)\cdot\sin\left(\frac{\pi}{b}y\right)\cdot\cos\left(\frac{\pi}{b}y\right) = 0$$

From here we obtain critical points (at these points the condition for the maximum of the function is satisfied $\rho_{2,1}(x,y)$):

$$x_1 = \frac{a}{4},\ y_1 = \frac{b}{2};\quad x_2 = \frac{3a}{4},\ y_2 = \frac{b}{2}.$$

To check the correctness of the problem, we compiled a program and selected the values of the sides of the potential well $a, b\ (b = 2a)$ and the quantum numbers $n_x = 2$ and $n_y = 1$ (Fig. 3. a). As a result, a probability density graph is displayed (Fig. 3. b). Figure 3.c shows a graph of the probability density in the plane. From Fig. 3. b, c it is clear that the probability density of a particle in a rectangular potential well reaches its maximum value at the points:

$$x_1 = \frac{a}{4},\ y_1 = \frac{b}{2}; \quad \text{and} \quad x_2 = \frac{3a}{4},\ y_2 = \frac{b}{2} \quad \text{(red area)}$$

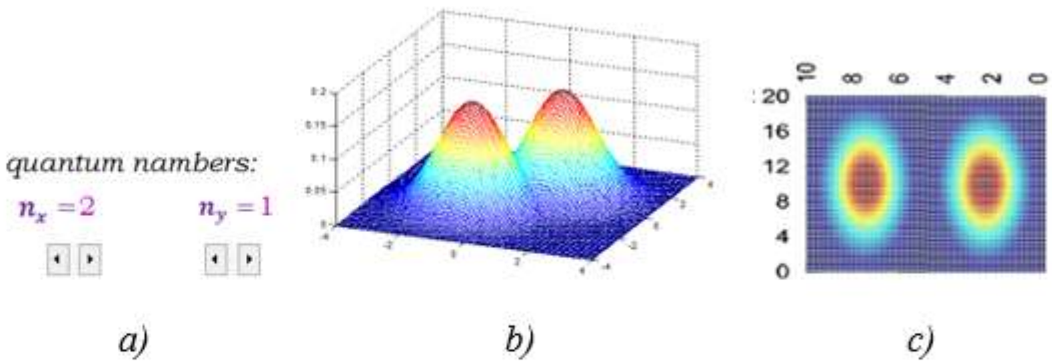

Fig.3. General view of the calculation results for the given conditions.
a-quantum numbers; b-graph of probability density; c-graph of probability density in the plane xOy.

2. The particle is in a two-dimensional rectangular potential well with infinitely high walls. The coordinates x, y of the particle lie in the range $0 < x < a,\ 0 < y < b,$ where a and $b -$ the sides of the pit (nm). Find the probability of finding a particle in the region $0 < x < a,\ 0 < y < b/4 \quad (b = 2a),$ at quantum numbers $n_x = 1,\ n_y = 2.$

The probability of finding a particle in two-dimensional space is defined by the expression:

$$\omega = A^2 \int_{x_1}^{x_2} \int_{y_1}^{y_2} \left|\psi_{n_x, n_y}(x, y)\right|^2 dxdy$$

If we insert the expression of the wave function into this formula, we obtain the following formula:

$$\omega = \frac{4}{ab} \int_{x_1}^{x_2} \int_{y_1}^{y_2} \sin^2\left(\frac{\pi n_x}{a} x\right) \cdot \sin^2\left(\frac{\pi n_y}{b} y\right) dxdy$$

Calculating the definite integral we obtain:

$$\omega_1 = \frac{4}{ab}\int_0^{a/}\sin^2\left(\frac{\pi}{a}x\right)dx \cdot \int_0^{b/4}\sin^2\left(\frac{2\pi}{b}y\right)dy = 0{,}25.$$

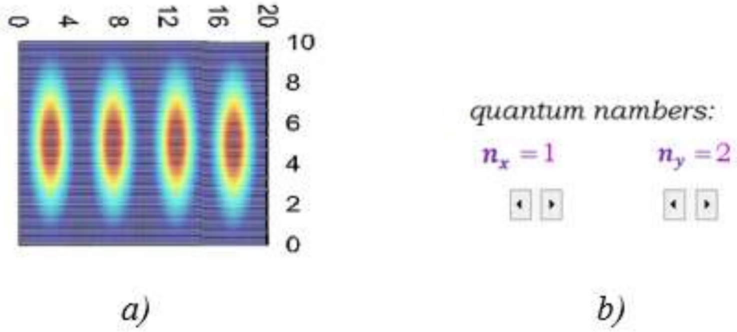

Figure 4. General view of the calculation results for the given conditions.
a – graph of probability density in the plane xOy; *b – quantum numbers;*

According to symmetry, the probability of finding a particle in the regions $0 < x < a$, $b/4 < y < b/2$; $0 < x < a$, $b/2 < y < 3b/4$; and $0 < x < a$, $3b/4 < y < b$; (Fig. 4) has the value respectively $\omega_2 = 25\%$, $\omega_3 = 25\%$, $\omega_4 = 25\%$. Then the probability of finding a particle in regions $0 < x < a$, $0 < y < b$ in a two-dimensional rectangular potential box with infinitely high walls takes the form:

$$\omega_1 + \omega_2 + \omega_3 + \omega_4 = 1$$

This satisfies the condition of finiteness of the wave function.

The above examples show the correctness of the created program and can be used to teach a two-dimensional potential well in quantum mechanics.

Findings

Such programs give an opportunity to visualize quantum phenomena and help deep understanding of the processes involved. This program can be used in teaching theoretical material and in solving problems in quantum mechanics as a demonstrative source.

CONCLUSION

In this work, we developed a computer simulation of a system with a particle in two-dimensional space at the quantum level with the corresponding software implementation and visualization of the results. Such concepts of quantum mechanics as the wave function and the probability density of particle detection are described in detail, and a new approach to the study of such problems in theoretical mechanics is formed.

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