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Calculation of Generalized Localization for Elliptic Partial Sums of Double Fourei Series in "Maple" Program

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Abstract. In this article, generalized localization for elliptic partial sums of double Fourier series was studied.

Key words: double Fourier series, generalized localization, Jacobi matrix.

For the one-dimensional case, in 1915 N.N. Luzin, in his dissertation called "Integral and trigonometric series", presented the hypothesis that the Fourier series of an arbitrary function in the class is almost approximate. For many years, Luzin's hypothesis attracted the attention of specialists who conducted several studies. Only by 1966, Carleson gave a positive answer to this hypothesis, that is, he proved that the Fourier series of the arbitrary function in the class almost converges. Carleson's result was strengthened by Hunt, that is, he proved that the Fourier series of an arbitrary function in it almost converges. On the other hand, in 1922 A.N. Kolmogorov gave an example of a function belonging to the class $L_1(T)$ such that the Fourier series of this function diverges almost even at every point.

The main goal of this work is to study the approximation of the partial sum (2). One of the questions that arises in the process of studying the near approximation of the sum (2) is whether Luzin's hypothesis is appropriate for the case we are considering: does $\forall f \in L_2\left(T^2\right)$ approximate the Fourier series of the function (2) in the circular sum? In other words, can Carleson's theorem be transferred to the circle sum of a double Fourier series. The only idea is that Hunt's theorem does not hold for the case we are considering [1]. The problem of solving Luzin's hypothesis began by solving simpler problems. One of these problems is that the partial sum of (2) almost approaches 0 in the set $T^2 \setminus \mathrm{supp} f$.

V. A. Ilin [2] introduced the concept of the generalized localization principle for an arbitrary distribution of eigenfunctions. Based on Ilin's concept, we say that the principle of generalized localization for $S_\lambda f(x,y)$ in $L_2(T^2)$ is valid if $\forall f \in L_2(T^2)$ is this for the function

$$\lim_{\lambda\to\infty} S_\lambda f(x,y)=0 \qquad (3)$$

if the equation is almost fulfilled.

It can be seen that, contrary to the classical localization principle, it is sufficient to fulfill equality (3) in $T^2 \setminus \mathrm{supp} f$ almost (but not everywhere).

The problem of generalized localization for the partial sum of N-fold Fourier series on a sphere was first studied by R.R. Ashurov [3] and a positive result was obtained in this regard.

Let's look at the Fure series below.

$$\sum_{n=-\infty}^{+\infty}\sum_{m=-\infty}^{+\infty} f_{nm}e^{i(nx+my)} \qquad (1)$$

Here $f_{nm}=(2\pi)^{-2}\int_{-\pi}^{\pi}\int_{-\pi}^{\pi} f(x,y)e^{-i(nx+my)}dxdy$ – Fourier coefficients.

We define the square by $T^2=(-\pi;\pi]\times(-\pi;\pi]$, let $S_\lambda f(x,y)$ – (1) be the elliptic partial sum of the series $[4]$, that is:

$$S_\lambda f(x,y)=\sum_{n^4+m^4<\lambda} f_{nm}e^{i(nx+my)}, \qquad (2)$$

here $\lambda>0$ is an arbitrary number. Let $L_2(T^2)$ – T^2 be the class of square-integrable functions. In this class, the norm can be entered as follows:

$$\|f\|_{L_2(T^2)}=\left(\int_{-\pi}^{\pi}\int_{-\pi}^{\pi}|f(x,y)|^2 dxdy\right)^{\frac{1}{2}}.$$

The main result of this paper is the following theorem.

Theorem 1. If:

1) $f\in L_2(T^2)$ *is periodic for each of its arguments, let the period be equal* 2π *;*

2) $\forall(x,y)\in\Omega$ ∂a $f(x,y)=0$*, is an open set, then* $\Omega\subset T^2$ *is an open set here.*

In that case, this

$$\lim_{\lambda\to\infty} S_\lambda f(x,y)=0 \qquad (4)$$

the relation is fulfilled almost everywhere in Ω *(that is, the dimension of the set of points in* Ω *for which equality (4) does not hold is equal to 0).*

When studying almost approximation problems, it is appropriate to include the maximum operator:

$$S_* f(x,y) = \sup_{\lambda>0} \left| S_\lambda f(x,y) \right|$$

The proof of Theorem 1 is based on the following estimate of the maximum operator:

Theorem 2. Let $K_R = \{(x,y):\ x^4 + y^4 < R\}$ *–be an elliptic sphere. if:*

1) $f(x,y) \in L_2(T^2)$*;*

2) $\forall (x,y) \in K_R \subset T^2 \partial a$ $f(x,y) = 0$*, then for* $\forall r\ (r < R)$*,* $\exists C = C(R,r)$ *There is an invariant that this*

$$) \int_{K_r} \left[S_* f(x,y) \right]^2 dxdy \le C \int_{T^2} \left[f(x,y) \right]^2 dxdy \qquad (5)$$

the inequality becomes relevant.

Now we use Newton's method to solve a system of nonlinear equations with two unknowns.

For this we get the following system of nonlinear equations.

$$\begin{cases} f_1(x_1, x_2, \ldots, x_n) = 0, \\ f_2(x_1, x_2, \ldots, x_n) = 0, \\ \quad \ldots \quad \ldots \quad \ldots \\ f_n(x_1, x_2, \ldots, x_n) = 0. \end{cases} \qquad (5)$$

We use the method of successive approximation (iteration) to find the solution of this system. We write the p-approximation to the solution of this sequence as follows:

$$x^{(p+1)} = x^{(p)} - W^{-1}\left(x^{(p)}\right) f\left(x^{(p)}\right) \qquad (6)$$

In this formula:

- $x^{(p)} = \left(x_1^{(p)},\ x_2^{(p)}, \ldots,\ x_n^{(p)}\right)$- denotes initial or π-approximation;

-$W^{-1}\left(x^n\right)$ (5) The following Jacobi matrix composed of the numbers found by the p-approximation value of the 1st-order eigenderivatives obtained for each argument of the functions on the left side of the system N2

$$W = \begin{pmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_n} \\ \cdots & \cdots & \cdots & \cdots \\ \frac{\partial fn}{\partial x_1} & \frac{\partial fn}{\partial x_2} & \cdots & \frac{\partial fn}{\partial x_n} \end{pmatrix} = \frac{\partial f_k}{\partial x_i},\ k,i = 1,2,3,\ldots,n \qquad (7)$$

is the inverse matrix of;

- $f\left(x^{(n)}\right)$ (5) is a matrix composed of the values of the functions on the left side of the system in $x^{(n)}$.

(6) the main condition for the sequence to converge to the solution:

$$\sum_{i=1}^{n}\left|\frac{\partial f_k}{\partial x_i}\right|<1,\quad k=1,2,...,\ n$$

I. Statement of the issue.

The following function $z=\sqrt{x^2+y^2}$

$$(x-m)^4+(y-n)^4=k$$

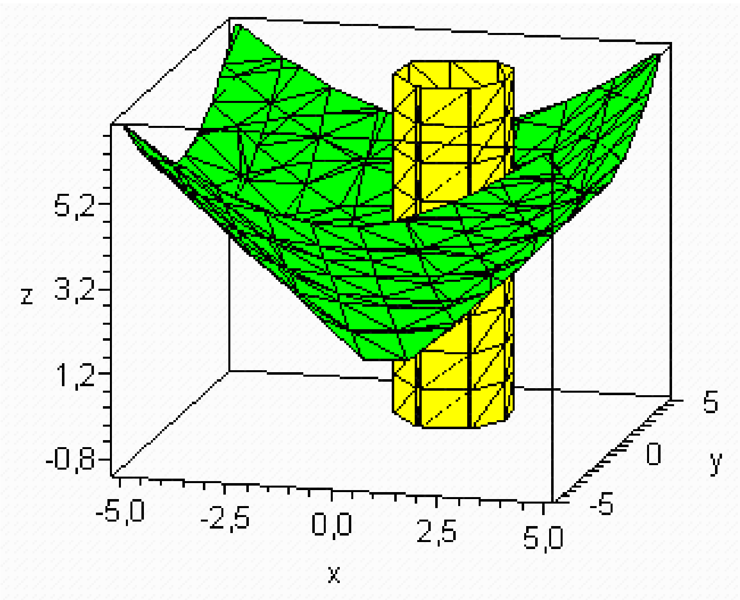
find the minimum value that satisfies the condition. Here $(n,m)\in Z^2$, $k\in N$.

II. Problem solving

$z(x,y)=\sqrt{x^2+y^2}$

$\varphi(x,y)=(x-m)^4+(y-n)^4-k$

We choose numbers as follows: $m=1$, $n=2$, $k=5$. Given $z=\sqrt{x^2+y^2}$ and $\varphi(x,y)=(x-1)^4+(y-2)^4-5$ their intersection graph:



Let's construct the Lagrangian function: $F=\sqrt{x^2+y^2}+\lambda\left[(x-1)^4+(y-2)^4-5\right]$

$$\frac{\partial F}{\partial x}=\frac{x}{\sqrt{x^2+y^2}}+4\lambda(x-1)^3;$$

$$\frac{\partial F}{\partial y}=\frac{y}{\sqrt{x^2+y^2}}+4\lambda(y-1)^3;\frac{\partial F}{\partial \lambda}=(x-1)^4+(y-2)^4-5$$

$$\begin{cases} x+4\lambda(x-1)^3\sqrt{x^2+y^2}=0 \\ y+4\lambda(y-1)^3\sqrt{x^2+y^2}=0; \\ (x-1)^4+(y-2)^4-5=0 \end{cases}$$

We find the solution of this nonlinear system to the first approximation by the iteration method:

$$F_1=x+4\lambda(x-1)^3\sqrt{x^2+y^2}=0$$

$$F_2=y+4\lambda(y-1)^3\sqrt{x^2+y^2}=0$$

$$F_3=(x-1)^4+(y-2)^4-5=0$$

The Jacobi W matrixes are constructed:

$$W=\left[1+12\lambda(x-1)^2\sqrt{x^2+y^2}+\frac{4\lambda(x-1)^3x}{\sqrt{x^2+y^2}},\frac{4\lambda(x-1)^3y}{\sqrt{x^2+y^2}},4(x-1)^3\sqrt{x^2+y^2}\right],$$

$$\left[\frac{4\lambda(y-2)^3x}{\sqrt{x^2+y^2}},1+12\lambda(y-2)^2\sqrt{x^2+y^2}+\frac{4\lambda(y-2)^3y}{\sqrt{x^2+y^2}},4(y-2)^3\sqrt{x^2+y^2}\right]$$

$$\left[4(x-1)^3,4(y-2)^3,0\right].$$

Jacobi matrix based on initial approximation values:

$$W\left(x^0\right)=\begin{pmatrix} 1.8839 & -1.7678 & -3.5356 \\ -4.7728 & 5.7732 & -9.5460 \\ -5.00 & -13.500 & 0 \end{pmatrix};$$

$$\det\left(W\left(x^{(0)}\right)\right)=\begin{vmatrix} 1.8839 & -1.7678 & -3.5356 \\ -4.7728 & 5.7732 & -9.5460 \\ -5.00 & -13.500 & 0 \end{vmatrix}=-267.42$$

$W(x^{(0)})$ we find the matrix inverse of the matrix

$$W^{-1}\left(x^{(0)}\right)=\begin{pmatrix} 0,48190 & -0.017848 & -0.013943 \\ -0.017848 & 0.0006104 & -0.073558 \\ -.25173 & -0.095432 & -0.037515 \end{pmatrix}$$

Based on the successive approximation formula, we find the values of the 1st approximation:

$$x^{(0)}=\begin{pmatrix} 0.5 \\ 0.5 \\ 0.5 \end{pmatrix},\ f\left(x^{(0)}\right)=\begin{pmatrix} -0.32322 \\ -4.2728 \\ -0.1250 \end{pmatrix}$$

$$x^{(1)} = x^{(0)} - W^{-1}\left(x^{(0)}\right) f\left(x^{(0)}\right) =$$

$$=\begin{pmatrix} 0.5 \\ 0.5 \\ 0.5 \end{pmatrix}-\begin{pmatrix} 0,48190 & -0.017848 & -0.013943 \\ -0.017848 & 0.0006104 & -0.073558 \\ -.25173 & -0.095432 & -0.037515 \end{pmatrix}\begin{pmatrix} -0.32322 \\ -4.2728 \\ -0.1250 \end{pmatrix}=\begin{pmatrix} 0.26972 \\ 0.51779 \\ 0.17829 \end{pmatrix}$$

We check the extremum at the first approximation point found in the iteration method above:

Based on the 1st approximation values x1=,

$$x^{(1)}=\begin{pmatrix} x \\ y \\ z \end{pmatrix}=\begin{pmatrix} 0.26972 \\ 0.51779 \\ 0.17829 \end{pmatrix}$$

we calculate the above:

$$\Delta_1=\begin{vmatrix} 0 & -1.557859231 & -13.02534419 \\ -1.557859231 & 2.488265867 & -.7017971716 \\ -13.02534419 & -.7017971716 & 5.065893532 \end{vmatrix}=462.933964$$

$$z_{\min}=0.58382819$$

We check the extremum at point

$$x^{(2)}=\begin{pmatrix} x \\ y \\ z \end{pmatrix}=\begin{pmatrix} 3.26972 \\ 3.51779 \\ -1.17829 \end{pmatrix}.$$

2- found in the above iteration method:

$$\Delta_1 = \begin{vmatrix} 0 & 46.77102040 & 13.98604924 \\ 46.77102040 & -72.72964562 & -.1038303774 \\ 13.98604924 & -.1038303774 & -32.47642057 \end{vmatrix} = -85133.8660$$

$$z_{\max} = 4.80269875$$

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